

The image shows a document viewer window. At the top, there's a header bar with a logo on the left, the text 'Ausschnitt 1' in the center, and 'Ausschnitt 2' on the right. Below the header, the document content is displayed. The first visible section is titled 'PAUSE ORIENTATION' in bold. The text below is in German and is heavily blurred. It appears to be a list or a series of points, with some words like 'Menschen', 'Kultur', 'Werte', 'Normen', 'Sitten', 'Gebräuche', 'Traditionen', 'Rituale', 'Feste', 'Feiern', 'Feste', 'Feiern', 'Feste', 'Feiern' being discernible. At the bottom of the document, there is a footer with the date '17-03-2009' and the file name 'My document'.

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1. **Lambert function**
Let f be a function from $\mathbb{R}$ to $\mathbb{R}$ such that $f(x) = x + f(x)^2$ for all $x \in \mathbb{R}$. Prove that f is bounded on $\mathbb{R}$.
Hint: Consider the function $g(x) = f(x) + 1$. Show that $g(x) \geq 1$ for all $x \in \mathbb{R}$. Then, use the fact that $g(x) = x + g(x)^2 - 1$ to show that $g(x)$ is bounded on $\mathbb{R}$. This implies that f is bounded on $\mathbb{R}$.
2. **Continuity of a function**
Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) = x + f(x)^2$ for all $x \in \mathbb{R}$. Prove that f is continuous on $\mathbb{R}$.
Hint: Use the fact that f is bounded on $\mathbb{R}$ (from the previous problem) to show that f is continuous on $\mathbb{R}$.
3. **Fixed point theorem**
Let $f: [0, 1] \rightarrow [0, 1]$ be a continuous function. Prove that f has a fixed point, i.e., there exists $x \in [0, 1]$ such that $f(x) = x$.
Hint: Consider the function $g(x) = f(x) - x$. Show that g has a root in $[0, 1]$.
4. **Intermediate value theorem**
Let $f: [a, b] \rightarrow \mathbb{R}$ be a continuous function. Prove that f takes on every value between $f(a)$ and $f(b)$.
Hint: Use the fact that f is continuous on $[a, b]$ to show that f is surjective onto the interval $[f(a), f(b)]$.
5. **Mean value theorem**
Let $f: [a, b] \rightarrow \mathbb{R}$ be a function such that $f(a) = f(b)$. Prove that there exists $c \in (a, b)$ such that $f'(c) = 0$.
Hint: Use the fact that f is continuous on $[a, b]$ and differentiable on (a, b) to apply the mean value theorem.
6. **Derivative of a function**
Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) = x + f(x)^2$ for all $x \in \mathbb{R}$. Prove that f is differentiable on $\mathbb{R}$ and find $f'(x)$.
Hint: Use the fact that f is continuous on $\mathbb{R}$ (from the previous problem) to show that f is differentiable on $\mathbb{R}$.
7. **Limit of a function**
Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) = x + f(x)^2$ for all $x \in \mathbb{R}$. Prove that $\lim_{x \rightarrow \infty} f(x) = \infty$.
Hint: Use the fact that f is bounded on $\mathbb{R}$ (from the previous problem) to show that f is unbounded on $\mathbb{R}$.
8. **Continuity of a function**
Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function such that $f(x) = x + f(x)^2$ for all $x \in \mathbb{R}$. Prove that f is continuous on $\mathbb{R}$.
Hint: Use the fact that f is bounded on $\mathbb{R}$ (from the previous problem) to show that f is continuous on $\mathbb{R}$.
9. **Fixed point theorem**
Let $f: [0, 1] \rightarrow [0, 1]$ be a continuous function. Prove that f has a fixed point, i.e., there exists $x \in [0, 1]$ such that $f(x) = x$.
Hint: Consider the function $g(x) = f(x) - x$. Show that g has a root in $[0, 1]$.
10. **Intermediate value theorem**
Let $f: [a, b] \rightarrow \mathbb{R}$ be a continuous function. Prove that f takes on every value between $f(a)$ and $f(b)$.
Hint: Use the fact that f is continuous on $[a, b]$ to show that f is surjective onto the interval $[f(a), f(b)]$.

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